To-Do
Math
English
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Math
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Physics

10/14 Lecture Notes

$\mathbb{R}^2$ = vector w/ 2 entries

$\mathbb{R}^2 + \mathbb{R}^3$ can't exist

magnitude & direction not good?

Linear transformations = how to transfer from $\mathbb{R}^2$ to $\mathbb{R}^3$ or $\mathbb{R}^{\text{whatever}}$

Vectors

addition: $\vec{u}+\vec{v} = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ \vdots \\ u_n+v_n \end{bmatrix}$

Ex] $2\begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix} + (-3)\begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 14 \\ -2 \end{bmatrix} + \begin{bmatrix} -15 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} -13 \\ 20 \\ -11 \end{bmatrix}$

scalar mult. $c\vec{u} = \begin{bmatrix} cu_1 \\ cu_2 \\ cu_3 \end{bmatrix}$

Properties: Let a, b, be scalars & $\vec{u}, \vec{v}, \vec{w}$ be vectors in $\mathbb{R}^n$

research cafe 1st Friday of the Month

a) $\vec{u}+\vec{v} = \vec{v}+\vec{u}$

b) $a(\vec{u}+\vec{v}) = a\vec{u}+a\vec{v}$

c) $(a+b)\vec{u} = a\vec{u}+b\vec{u}$

d) $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$

e) $a(b\vec{u}) = (ab)\vec{u}$

f) $u+(-\vec{u}) = \vec{0}$ (0 vector)

g) $\vec{u}+\vec{0} = \vec{u}$

h) $1\cdot\vec{u} = \vec{u}$

If $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_M$ are vectors with the same # of entries in $\mathbb{R}^N$ and $c_1, \ldots, c_M$ are scalars, then $c_1\vec{u}_1 + \ldots + c_M\vec{u}_M$ is called a linear combination of the vectors

Ex] I have two solutions; one is 10% sugar, 10% salt, and 2% iron. The other is 5% sugar, 12% salt, and 3% salt. How much of each solution is required to get a liquid w/ 20 liters of sugar, 34 liters of salt and 8 liters of iron?

Amount of solution A is x_1 & amount of solution B is x_2.

$x_1 = \begin{bmatrix} .1 \\ .1 \\ .02 \end{bmatrix} = \begin{bmatrix} \text{liters of sugar} \\ \text{" salt} \\ \text{" iron} \end{bmatrix}$ $\quad x_2 = \begin{bmatrix} .05 \\ .12 \\ .03 \end{bmatrix}$ } If we combine these 2, $x_1 + x_2$,

$x_1 + x_2 = \begin{bmatrix} 20 \\ 34 \\ 8 \end{bmatrix} \Longrightarrow \begin{bmatrix} .1x_1 + .05x_2 \\ .1x_1 + .12x_2 \\ .02x_1 + .03x_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 34 \\ 8 \end{bmatrix}$ so aug. matrix $= \left[\begin{array}{cc|c} .1 & .05 & 20 \\ .1 & .12 & 34 \\ .02 & .03 & 8 \end{array}\right]$

when solved: $\left[\begin{array}{cc|c} 1 & 0 & 100 \\ 0 & 1 & 200 \\ 0 & 0 & 0 \end{array}\right] \Longrightarrow x_1 = 100,\ x_2 = 200$ $\quad \vec{x} = \begin{bmatrix} \text{amt of A} \\ \text{amt of B} \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \end{bmatrix}$ } vector form

Q: can it get to $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$? & can it get to all pts in $\mathbb{R}^3$?

Ex] I have a robot that can move in two diff. ways, the same distance & direc. in the x direction as the y-direction. Or, it can go in the same distance & direction in the y-direction as the z-direction. Assume origin is (0,0,0), we can describe where the robot goes as: $x_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ } amt x is same as amt y $+ x_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ } amt y is same as amt z

(a) $\Rightarrow x_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 5 \end{bmatrix} \Rightarrow$ a.m. $= \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 7 \\ 0 & 1 & 5 \end{array}\right] \xrightarrow{-R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right]$

so answer to (a) is yes, going 2 units of x_1 (1st move) and 5 units of second move x_2 will get to $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

Robot cannot get to all pts.
Span = set of all lin. eq's.